聖公會林護紀念中學

2022-23 中一學位申請 第一階段面試 (數學解難)

偉明在電腦玩一個估數字遊戲。電腦從 1 至 9 中隨機抽取一個整數 (**不包括** 1 及 9)，然後偉明猜這個整數。如果偉明猜中，電腦會告訴偉明猜中了。如果猜不中，電腦會告訴偉明，電腦選的數字是大於還是小於偉明選的數字。

以下是一個遊戲例子：

電腦：請猜我的數字 6 電腦：錯誤，我的數字少於 6。請猜我的數字。 4 電腦：錯誤，我的數字少於 4。請猜我的數字。 2 電腦：錯誤，我的數字大於 2。請猜我的數字。 3 電腦：正確。我的數字是 3。你用了 4 次猜中。

1. 如果偉明希望保證在 n 次之內必定猜中，而 n 要盡量小。則
   (a) 偉明第一次應該猜哪一個數字？ (1 分)
   (b) n 的最小可能值是甚麼？ (1 分)
   (c) 如果規則改為電腦從 1 至 99 中隨機抽取一個整數(不包括 1 及 99)，則 n 的最小可能值是甚麼？ (2 分)

2. 現在遊戲再進行變化：電腦從 1 至 10 中隨機抽取一個整數 (不包括 1 及 10)，然後偉明可以猜兩個整數 (設兩數為 a 及 b, 其中 b 大於 a)。如果偉明猜的兩個數之中，有任何一個猜中，電腦會告訴偉明猜中了。如果偉明猜的兩個數皆猜不中，電腦會告訴偉明，電腦選的數字是大於 b, 小於 a, 還是在 a 和 b 之間。

   如果偉明希望保證在 n 次之內必定猜中，而 n 要盡量小。則
   (a) (i) 偉明第一次應該猜哪兩個數字？ (1 分)
   (ii) n 的最小可能值是甚麼？ (1 分)
   (b) 如果規則改為電腦從 1 至 X 中隨機抽取一個整數 (不包括 1 及 X), 而且保證偉明可以在 4 次之內必定猜中，則 X 的最大可能值是甚麼？你如何計算出這值的？ (3 分)

(應考同學在面試作答時，不需重複讀問題。另外，面試老師可能對你的解答作出追問)。

S.K.H. Lam Woo Memorial Secondary School

2022-2023 F.1 Admission Application

First Round Interview (Problem Solving in Mathematics) Set A

Ken is playing a computer game about number guessing. The computer randomly picks an integer from 1 to 9 (**excluding** 1 and 9), then Ken guesses the number. If Ken guesses it correctly, the computer will tell Ken that he is correct. If Ken does not guess it correctly, the computer will tell Ken whether the computer's number is greater than or smaller than the number that Ken guessed. Here is an example:

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Computer: Please guess my number.
6
Computer: Wrong, my number is smaller than 6. Please guess my number.
4
Computer: Wrong, my number is smaller than 4. Please guess my number.
2
Computer: Wrong, my number is greater than 2. Please guess my number.
3
Computer: Correct, my number is 3. You guessed correctly in 4 times.
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1. If Ken wants to minimize n so that he can guarantee that he must guess the computer's number correctly within n times. Then
   (a) Which number should Ken guess for the first time? (1 mark)
   (b) What is the minimum possible value of n? (1 mark)
   (c) If the rule is now changed so that the computer randomly picks an integer from 1 to 99 (excluding 1 and 99), then what is the minimum possible value of n? (2 marks)
2. The game is now changed further: The computer randomly picks an integer from 1 to 10 (excluding 1 and 10), then Ken can guess two integers. (Let the two integers be a and b, where b is greater than a). If any one of the two numbers Ken guesses equals to the computer's number, the computer will tell Ken that he is correct. If none of the two numbers Ken guesses equals to the computer's number, the computer will tell Ken whether the computer's number is greater than b, smaller than a, or between a and b.
   If Ken wants to minimize n so that he can guarantee that he must guess the computer's number correctly within n times. Then
   (a) (i) Which two numbers should Ken guess for the first time? (1 mark)
   (ii) What is the minimum possible value of n? (1 mark)
   (b) If the rule is now changed so that the computer randomly picks an integer from 1 to X (excluding 1 and X), and it can be guaranteed that Ken can guess the number correctly within 4 times, then what is the maximum possible value of X? How did you find this value? (3 marks)

(In the interview, candidates need not repeat the questions to the interviewers).

(The interviewers may ask you further question to elaborate your answer).